WUR/EEES 21306
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Homework 9&10
Due: Dec 6 (before class)
5 points per question
10 points total

9. Optimal pollution reductions with two firms (NB: Not my favorite choice of question, but later professors will ask you to do problems like this).

   Firm A has Total Abatement Cost (TAC) function of $C_A = 50 + 3Z_A^2$. Firm B has a TAC of $C_B = 75 + 2Z_B^2$. The government is targeting a total of Z=50 units of reduction.

   (a) (1 pt) What are the costs to A and B for $Z_A = Z_B = 25$ units?
   Answer: $C_A(25) = 50 + 3(25^2) = 1925$ and $C_B(25) = 75 + 2(25^2) = 1325$.

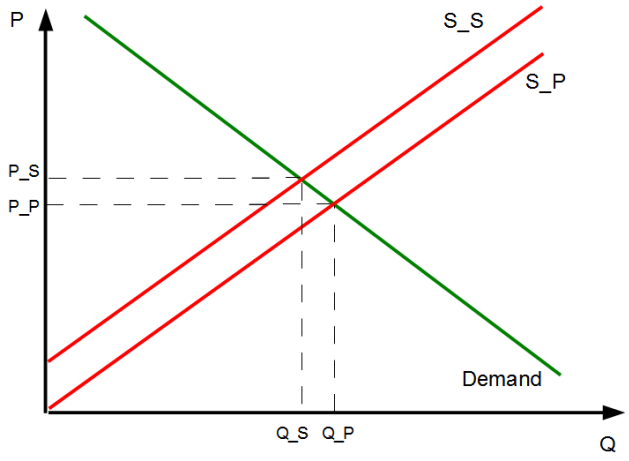
   (b) (1 pt) What are the values of Z_A and Z_B for a cost-effective reduction in pollution that minimizes total costs subject to $Z = 50$?
   Answer: Set up the Lagrangian function: $C_A + C_B - \lambda(Z_A + Z_B - 50) \Rightarrow 50 + 3Z_A^2 + 75 + 2Z_B^2 + \lambda(50 - Z_A - Z_B)$. Take FOCs to find Z_A^* and Z_B^* and MCs. From the Lagrangian FOCs, $6Z_A = 4Z_B$, so $Z_B = 1.5Z_A = 30$ and $Z_A = 20$.

   (c) (1 pt) What are the marginal costs for each? What are total costs?
   Answer: From the FOCs of $6Z_A = 4Z_B$, we know the marginal cost for both is 120. Total costs are 3125.

   (d) (2 pts) Now that you've done these calculations, tell me (three sentences maximum) what you'd say to the minister of the environment if he asked you to find the optimal amounts of pollution for two (or 40 or 4,000) Dutch firms.
   Answer: Something like "I don't know their TACs, so it's hard to find the right reductions for each. It would be better to set a tax on pollution or put a cap and allow trading." Answers along the lines of "find TACs and set the efficient tax" get zero points.

10. We will compare emissions taxes to permits for pollution from CO_2-emitting power plants, with and without perfect information. NB: Make a LARGE graph (one half A4) so you have enough space; make sure you label curves.

    (a) (1 pt) P is the price of power; Q is the quantity on the market. Draw a demand curve and supply curve that reflects the private cost of selling power; label it S_P. Now add a supply curve that includes the social cost of CO_2 emissions (make it parallel to S_P); label it S_S. On the vertical price axis, label equilibrium prices P_P for the private price and P_S for the social price. On the horizontal axis, label equilibrium quantities Q_P for the quantity sold when prices only reflect private costs and Q_S when they include social costs.
    Answer: Here's the picture:



(b) (0.5 pt) Does the imposition of an accurate Pigouvian tax first cause P_P to move to P_S or Q_P to move to Q_S?

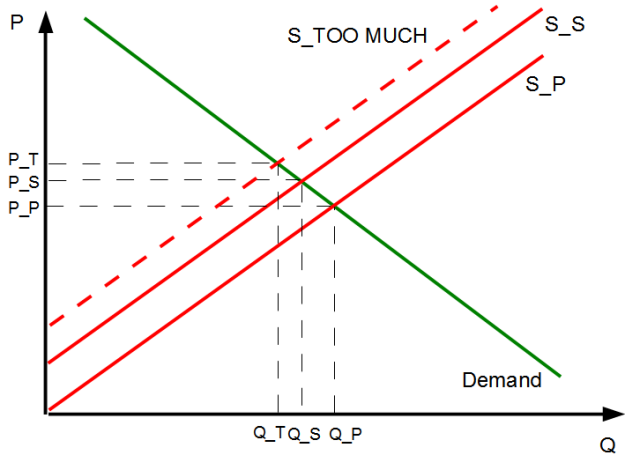
Answer: The tax of t increases P_P to P_S. The consumer, seeing this higher price, chooses to move from Q_P to Q_S.

(c) (0.5 pt)Does the imposition of an accurate limit on emissions (cap and trade) first cause P_P to move to P_S or Q_P to move to Q_S?

Answer: A quantity limit reduces Q_P to Q_S.[1]

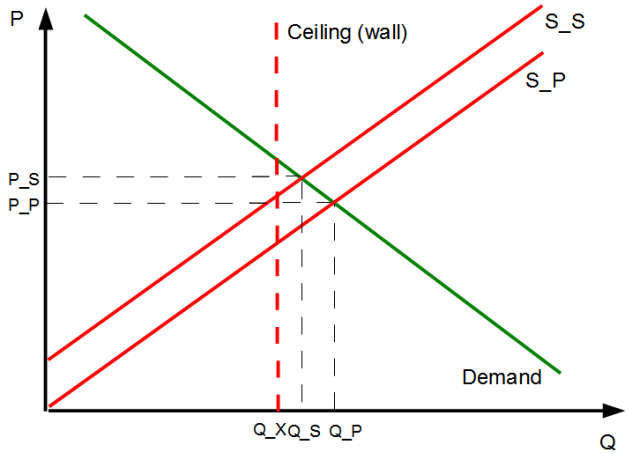
(d) (1.5 pts) Now assume that the tax is set too high. Draw a new supply curve, S_W. Is the quantity of power produced too high or too low, relative to Q_S? Who gains, within the power industry, from the tax being too high?

Answer: A tax that's too high will increase prices by *too much* and thereby reduce quantity by too far (see picture). Zero-carbon (e.g., wind, solar, nuclear) power generators will win in this case, since they can sell more power (probably at higher prices) to fill in the space left by the over-taxed carbon emitters.



(e) (1.5 pts) Now assume that the (cap and trade) permit ceiling is set too low. Draw a vertical line showing this limit. Label its intersection with the quantity axis Q_X. Who gains, within the CO_2 emitting industry, from permits being too scarce?

Answer: See picture for the low ceiling (more like a wall :). The winners within industry are net sellers of permits who make extra money from the excess demand for permits.



[1]FYI, trade will reallocate permits among owner-sellers and buyer-users, such that the cost of production rises to reflect the social cost (i.e., to P_S). Transaction costs will reduce the efficiency of this move (there are also TCs for taxes), so it's not easy to get to the theoretical optimum. Funds transferred from buyers to seller are ignored, since they represent a transfer among parties, not a change in aggregate wealth.