EEP 100 — Problem Set 3 (Due Nov 24, 2009)

(1) (1 pt) Find the mixed-strategy equilibrium (percent of the time for each move) for Player 1 and Player 2 in this simultaneous move, one-shot game. (Hint: Think reaction functions).

		Player 2	
		Left	Right
Player 1	Up	5,2	1,4
	Down	2,5	3,2

Solution. Player 1 will change positions until her expected profit (given Player 2's move) from up is equal to her expected profit from down. Player 2 knows this and can use it (because of complete information) to find his moves so he also has the same expected profit from each of his moves, given hers. In other words, Player 2 sets expected profits for each move (up or down) equal to each other.

Thus (using p_l for the probability that Player 2 moves left), we set expected profits for Player 1 moving up equal to expected profits from moving down. (Player 2 uses this information!)

$$\pi_u = 5p_l + 1(1 - p_l) \overset{set}{=} \pi_d = 2p_l + 3(1 - p_l)$$

Solving, we find that $p_l = 0.4$. p_r is therefore 0.6.

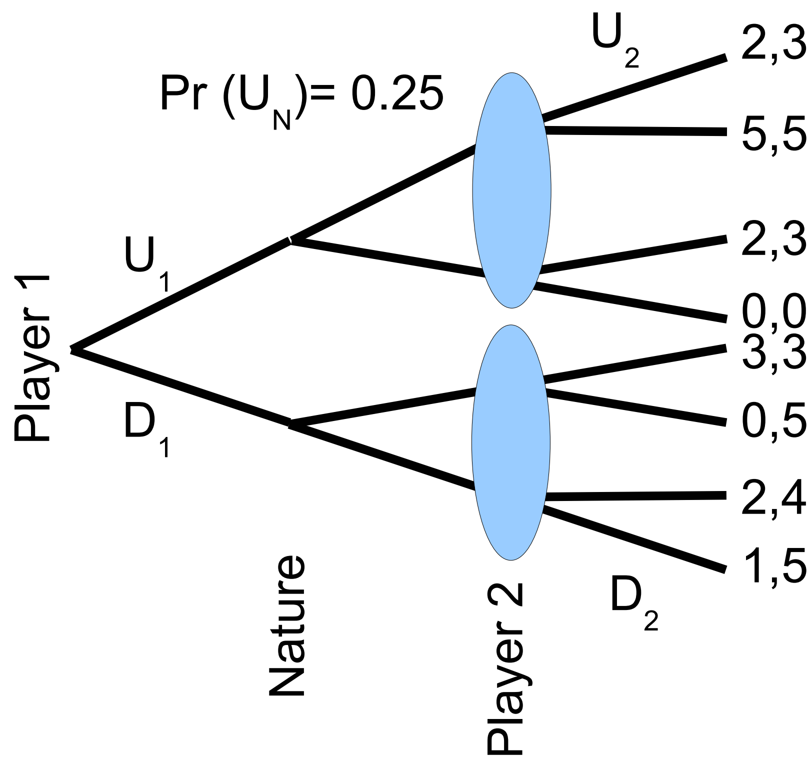
Player 1 looks at Player 2's expected profits to find her moves:

$$\pi_l = 2p_u + 5(1 - p_u) \overset{set}{=} \pi_r = 4p_u + 2(1 - p_u)$$

Solving, we find that $p_u = 0.6$. p_d is therefore 0.4.

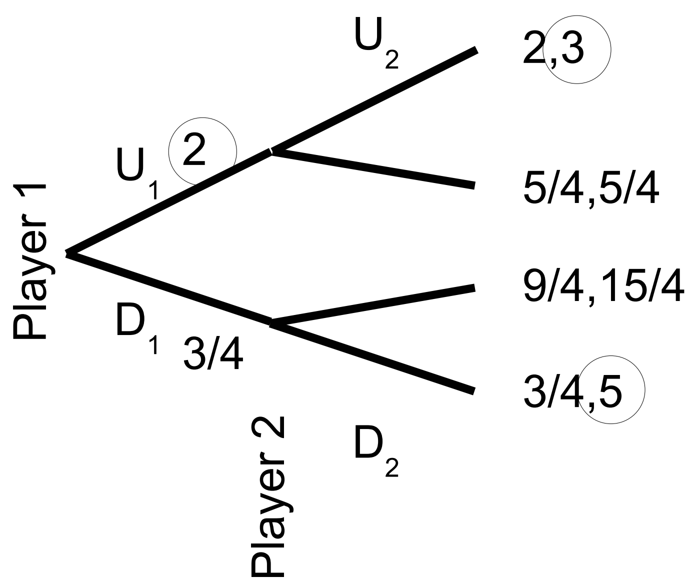
Note that Player 1's expected profit is 2.6 and Player 2's expected profit is 3.2. (This trivia has no strategic significance.)

(2) (1 pt) Find the strategy for Player 1 and Player 2 (what moves to make) in this sequential move, one-shot game. Note that Nature moves after Player 1, going UP with a probability of 0.25 and DOWN with a probability of 0.75. Neither player knows Nature's move. Thus, Player 2 observes Player 1's move but does not know if Nature goes up or down.



[NOTE: I did not label all branches with moves because there was no space, but your answer should be in the form of "Player 1 moves x; and Player 2 moves y" where x and y mean up or down.]

Solution. {Up, Up}. Use expected payoffs to calculate what each player should expect to get. Note that you have to add the "up" branch (25%) and down branch (75%) to get the expected payoff for Player 2's up move, given that Player 1 chose U as well. Thus, the expected payoff from D_2, given U_1, is $5(0.25) + 3(0.75) = 1.25$.



(3) (1.5 pts) Suppose there are two firms in an isolated market (no other entry) that produce identical beef. Firm 1s' marginal cost is \$1 per lb of beef, and Firm 2's marginal cost is \$2 per lb; there are no fixed costs. Suppose that each firm chooses its own output simultaneously without knowing its rival's choice (therefore this is a Cournot model). Market demand is $Q = 100 - 10P$.

(a) Find the reaction function for each firm, and draw them in a two-dimensional diagram with q_1 on the horizontal axis and q_2 on the vertical axis, where q_1 is Firm 1's output and q_2 is Firm 2's output.

(b) Find the equilibrium output for each firm using their reaction functions, and label equilibrium on your diagram from part (a).

(c) Find market price (P^*) and quantity (Q^*) in equilibrium.

(d) Compute each firm's equilibrium profit.

Solution.

(a) The inverse demand function is:

$$P = 10 - \frac{Q}{10} = 10 - \frac{q_1 + q_2}{10}$$

Firm 1's profit-max problem is:

$$\underset{q_1}{Max}\ \pi_1 = P \cdot q_1 - 1 \cdot q_1 = \left(10 - \frac{q_1 + q_2}{10}\right) q_1 - q_1 = \left(9 - \frac{q_1 + q_2}{10}\right) q_1 = 9q_1 - \frac{q_1^2}{10} - \frac{q_1 q_2}{10}$$

The first-order condition is:

$$\frac{\partial \pi_1}{\partial q_1} = 9 - \frac{q_1}{5} - \frac{q_2}{10} = 0 \implies q_1 = 45 - \frac{q_2}{2}$$

Therefore firm 1's reaction function is:

$$q_1 = R(q_2) = 45 - \frac{q_2}{2}$$

Firm 2's profit-max problem is:

$$\underset{q_2}{Max}\ \pi_2 = P \cdot q_2 - 2 \cdot q_2 = \left(10 - \frac{q_1 + q_2}{10}\right) q_2 - 2q_2 = \left(8 - \frac{q_1 + q_2}{10}\right) q_2 = 8q_2 - \frac{q_2^2}{10} - \frac{q_1 q_2}{10}$$

The first-order condition is:

$$\frac{\partial \pi_2}{\partial q_2} = 8 - \frac{q_2}{5} - \frac{q_1}{10} = 0 \implies q_2 = 40 - \frac{q_1}{2}$$

Therefore firm 2's reaction function is:

$$q_2 = R(q_1) = 40 - \frac{q_1}{2}$$

This figure shows the reaction functions for both firms.

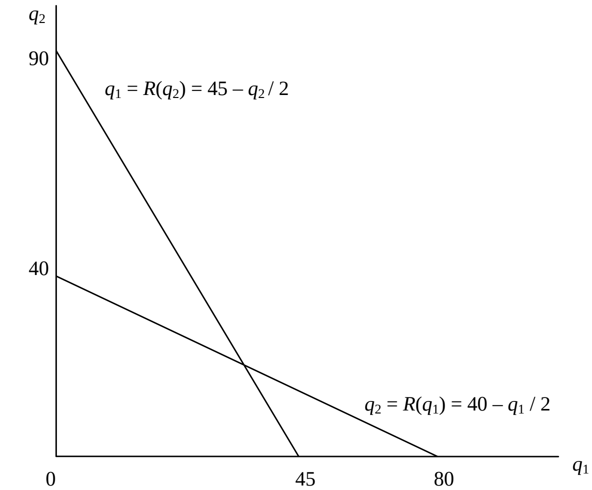


Figure 1. Best-response functions

(b) The reaction functions are:

$$\begin{cases} q_1 = 45 - \frac{q_2}{2} \\ q_2 = 40 - \frac{q_1}{2} \end{cases}$$

We can then compute the Cournot equilibrium $(q_1^\star,\ q_2^\star)$ by solving the two equations above:
$q_1 = 45 - \frac{q_2}{2} = 45 - \frac{1}{2}(40 - \frac{q_1}{2}) \implies q_1^\star = \frac{100}{3}$
$q_2^\star = 40 - \frac{q_1^\star}{2} = 40 - \frac{50}{3} = \frac{70}{3}$
Therefore the Cournot equilibrium output for each firm is:

$$\begin{cases} q_1^\star = \frac{100}{3} \\ q_2^\star = \frac{70}{3} \end{cases}$$

This figure shows the Cournot equilibrium point.

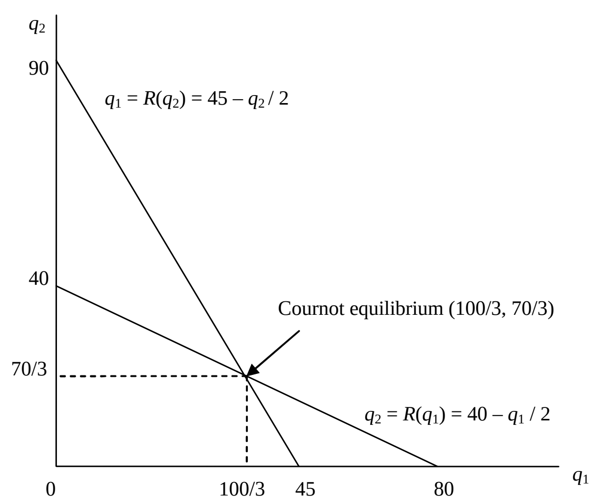


Figure 2. Cournot equilibrium

(c) Since Cournot equilibrium output for each firm is: $q_1^\star = \frac{100}{3}$, $q_2^\star = \frac{70}{3}$, therefore the total quantity supplied in the market is:

$$Q^\star = q_1^\star + q_2^\star = \frac{170}{3}$$

The price in the market is:

$$P^\star = 10 - \frac{Q^\star}{10} = 10 - \frac{17}{3} = \frac{13}{3}$$

(d) The optimal profit for firm 1 is:

$$\pi_1^\star = P^\star \cdot q_1^\star - 1 \cdot q_1^\star = (\frac{13}{3} - 1) \cdot \frac{100}{3} = \frac{1000}{9}$$

The optimal profit for firm 2 is:

$$\pi_2^\star = P^\star \cdot q_2^\star - 2 \cdot q_2^\star = (\frac{13}{3} - 2) \cdot \frac{70}{3} = \frac{490}{9}$$

(4) (1.5 pts) Consider a market of beef with the same setup as in the previous question *except* that Firm 1 is a Stackelberg leader that chooses its own output (q_1) first. Firm 2 (the follower) observes this choice before choosing its output (q_2.

(a) Find Firm 1's optimal action and Firm 2's optimal response. Draw them in a two-dimensional diagram with q_1 on the horizontal axis and q_2 on the vertical axis.

(b) Find the equilibrium output for each firm, and label the equilibrium point in your diagram in part (a).

(c) Find market price (P^*) and quantity (Q^*) in equilibrium. Is the price higher or lower than in Question 3? What about total quantity?

(d) Compute the profit for each firm in equilibrium. Is Firm 1's profit higher or lower than Firm 1's profit in Question 3? What about Firm 2's profit?

Solution.

(a) The inverse demand function is still:

$$P = 10 - \frac{Q}{10} = 10 - \frac{q_1 + q_2}{10}$$

Now in the Stackelberg game, the follower (firm 2)'s profit-max problem does not change, which is still:

$$\underset{q_2}{Max}\ \pi_2 = P \cdot q_2 - 2 \cdot q_2 = \left(10 - \frac{q_1 + q_2}{10}\right) q_2 - 2q_2 = \left(8 - \frac{q_1 + q_2}{10}\right) q_2 = 8q_2 - \frac{q_2^2}{10} - \frac{q_1 q_2}{10}$$

The first-order condition does not change either:

$$\frac{\partial \pi_2}{\partial q_2} = 8 - \frac{q_2}{5} - \frac{q_1}{10} = 0 \implies q_2 = 40 - \frac{q_1}{2}$$

Therefore the follower (firm 2)'s reaction function (or reaction function) is the same as in question 3, namely:

$$q_2 = R(q_1) = 40 - \frac{q_1}{2}$$

The leader (firm 1)'s profit-max problem is different because firm 1 now knows firm 2's reaction function and will incorporate that into its own profit-max problem:

$$\underset{q_1}{Max}\ \pi_1 = P \cdot q_1 - 1 \cdot q_1 = \left(10 - \frac{q_1 + R(q_1)}{10}\right) q_1 - q_1 = \left(9 - \frac{q_1 + 40 - \frac{q_1}{2}}{10}\right) q_1 = 5q_1 - \frac{q_1^2}{20}$$

The first-order condition is now:

$$\frac{\partial \pi_1}{\partial q_1} = 5 - \frac{q_1}{10} = 0 \implies q_1 = 50$$

Figure 3 shows firm 2's reaction function ($q_2 = 40 - \frac{q_1}{2}$) and firm 1's optimal action ($q_1^\star = 50$).

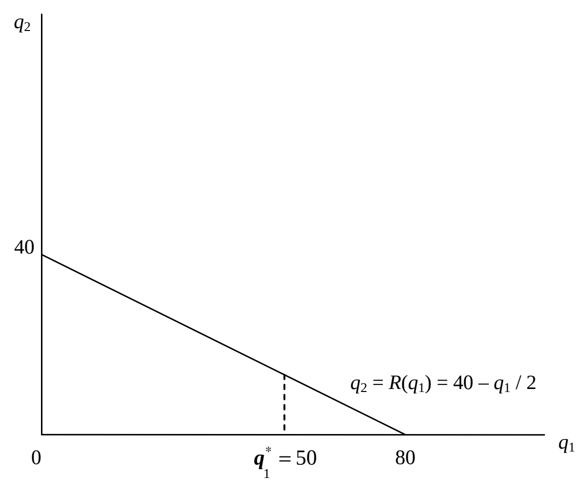


Figure 3. Follower's best-response function

(b) Since firm 1's optimal action is: $q_1^\star = 50$, then we can calculate firm 2's optimal action by substituting it into firm 2's reaction function:

$$q_2 = 40 - \frac{q_1}{2} = 40 - 25 = 15$$

Therefore the Stackelberg equilibrium output for each firm is:

$$\begin{cases} q_1^\star = 50 \\ q_2^\star = 15 \end{cases}$$

Figure 4 shows the Stackelberg equilibrium point $(q_1^\star,\, q_2^\star) = (50,\, 15)$.
This figure shows the Stackelberg equilibrium point.

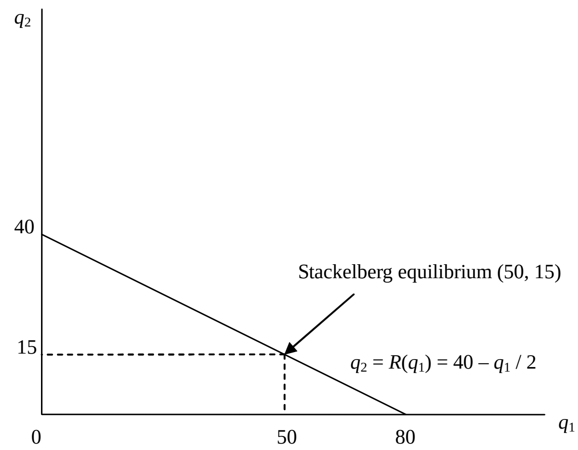


Figure 4. Stackelberg equilibrium

(c) Since Stackelberg equilibrium output for each firm is: $q_1^\star = 50$, $q_2^\star = 15$, then the market quantity is:

$$Q^\star = q_1^\star + q_2^\star = 50 + 15 = 65$$

Thus the market price is:

$$P^\star = 10 - \frac{Q^\star}{10} = 10 - \frac{65}{10} = 3.5 < \frac{13}{3}$$

Therefore the market price in the Stackelberg game is lower than in the Cournot game (question 3).

(d) The leader (firm 1)'s equilibrium profit is:

$$\pi_1^\star = P^\star \cdot q_1^\star - 1 \cdot q_1^\star = (3.5 - 1) \cdot 50 = 125 > \frac{1000}{9}$$

Therefore the leader's equilibrium profit is higher in the Stackelberg game than in the Cournot game (question 3).

The follower (firm 2)'s equilibrium profit is:

$$\pi_2^\star = P^\star \cdot q_2^\star - 2 \cdot q_2^\star = (3.5 - 2) \cdot 15 = 22.5 < \frac{490}{9}$$

Therefore the follower's equilibrium profit is lower in the Stackelberg game than in the Cournot game (question 3).