Due 9 Sep 2014. Turn in typed or neatly written answers.

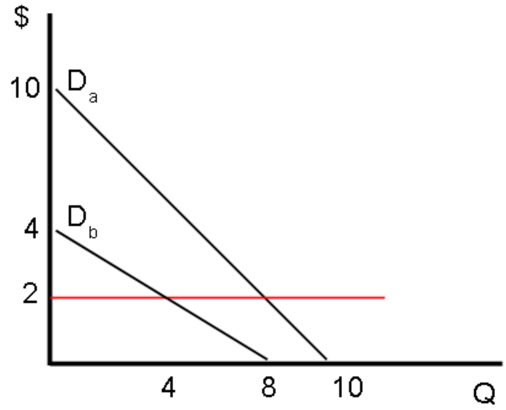
1. (4 points) Ana and Bob both like bananas. Their (inverse) demand curves are $Q_a = 10 - p$ and $Q_b = 8 - 2p$, respectively.

   (a) (1 point) Calculate their quantity demanded for bananas at $p = 2$.

   Solution: $Q_a = 10 - 2 = 8$ and $Q_b = 8 - 2(2) = 4$

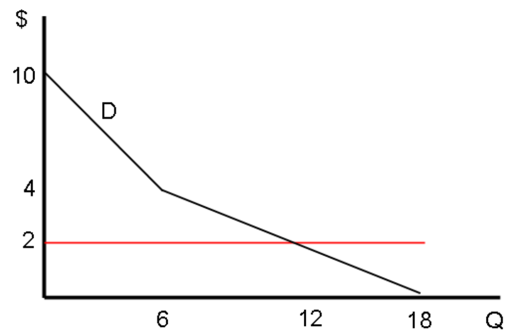
   (b) (1 point) Draw each of their inverse demand curves on one set of axes (p on the vertical axis). Show the quantity of bananas both consume.

   Solution: Figure with two demand curves



   (c) (1 point) Now draw their aggregate (inverse) demand curve and show their total quantity demanded, given $p = 2$

   Solution: The aggregate demand curve kinks at (q,p) = (6,4) and ends at Q=18, falling with a slope of $-\frac{1}{3}$ Q. Many people missed this, but you could explore the numbers to find the right shape.



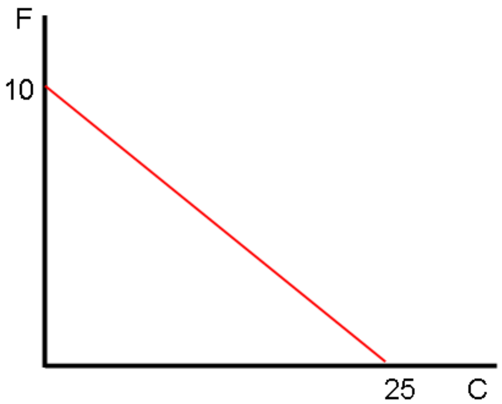
   (d) (1 point) You are a banana seller and observe Ana and Bob's banana purchases. Do you know if Ana or Bob likes bananas more? Explain.

   Solution: This was a trick question to highlight the difference between academic economics (I can see your demand curve) and the real world. As a banana vendor, you don't know who likes bananas *more* because you cannot observe customer utility or demand functions. You only see how many bananas each consumes without knowing their surplus or WTP. Ana, e.g., may have a flatter curve that means a lower surplus (utility) from a greater consumption of bananas.

2. (6 points) David Ricardo described "comparative advantage" in 1817, i.e., the potential for both sides to benefit from trade, even if one side is worse at producing both goods. We will use this idea here.

   (a) (1 point) You are stranded on a desert island, where you can catch 2 fish per hour or harvest 5 coconuts per hour. Under the tropical sun, you can only work 5 hours per day. Draw your "production potential" for fish (vertical axis) and coconuts (horizontal axis).

   Solution:

   (b) (3 points) You like fish more than coconuts but always some of each, i.e, $u(f,c) = f^{\frac{2}{3}}c^{\frac{1}{3}}$. Use the Lagrangian method to calculate your utility maximizing consumption on a normal day. Show all your steps AND double check that your final consumption exhausts your time (budget constraint).

   Solution:

$$\max u(f,c) = f^{\frac{2}{3}}c^{\frac{1}{3}} \text{ s.t. } 5 \geq \frac{1}{2}f + \frac{1}{5}c$$

$$\begin{aligned}
\mathcal{L} &: f^{\frac{2}{3}}c^{\frac{1}{3}} + \lambda(5 - \frac{1}{2}f - \frac{1}{5}c) \\
\frac{\delta\mathcal{L}}{\delta f} &: \frac{2}{3}f^{\frac{-1}{3}}c^{\frac{1}{3}} - \frac{1}{2}\lambda \stackrel{\text{set}}{=} 0 \\
\frac{\delta\mathcal{L}}{\delta c} &: \frac{1}{3}f^{\frac{2}{3}}c^{\frac{-2}{3}} - \frac{1}{5}\lambda \stackrel{\text{set}}{=} 0 \\
\frac{\delta\mathcal{L}}{\delta\lambda} &: 5 - \frac{1}{2}f - \frac{1}{5}c \stackrel{\text{set}}{=} 0
\end{aligned}$$

Now do some algebra to find c as a function of f using λ, i.e., $2(\frac{2}{3}f^{\frac{-1}{3}}c^{\frac{1}{3}}) = 5(f^{\frac{2}{3}}c^{\frac{-2}{3}}) \rightarrow$ (via multiplying both sides by $f^{\frac{1}{3}}c^{\frac{2}{3}}$) $\frac{4}{3}c = \frac{5}{3}f \rightarrow c = \frac{5}{4}f$.
Using this "optimal" ratio of f to c with the budget constraint, we get $5 = \frac{1}{2}f + \frac{1}{5}\frac{5}{4}f = \frac{3}{4}f \rightarrow f = \frac{20}{3}$ and $c = \frac{100}{12} = \frac{25}{3}$, which checks as $5 = \frac{1}{2}\frac{20}{3} + \frac{1}{5}\frac{25}{3}$!

(c) (1 point) One day, you meet Max, a friendly guy who is also stranded on the island. He is better at harvesting both fish and coconuts (at a rate of 3 and 12 per hour, respectively). Max offers to trade 10 coconuts for 3 of your fish (only once). How many fish and coconuts will you have before and after trading? Explain, using utility, if you should trade with Max.

Solution: Before trading, you can get $\frac{20}{3}f$ and $\frac{25}{3}c$. If you start a new day ("one day") by putting all your time into catching fish (i.e., 10) and then trade 3 fish for 10 coconuts, then you can get $7f$ and $10c$ (or $\frac{21}{3}f$, $\frac{30}{3}c$), which is better than you would do without trading.

(d) (1 point) How is it possible that Max can benefit from trading with you when he's better at harvesting BOTH fish and coconuts? Explain using the concept of "opportunity cost."

Solution: Max is better off because he can specialize in collecting coconuts while trading for fish. Without trade, he gives up 12 coconuts (the opportunity cost) to get 3 fish. With trading, he only needs to "spend" 10 coconuts to get 3 fish. Trade allows both sides to benefit from different opportunity cost ratios, i.e., their comparative advantages.